KU LEUVEN

11th World Congress on Computational Mechanics
WCCM XI – Barcelona, Spain
July 20 – 25, 2014

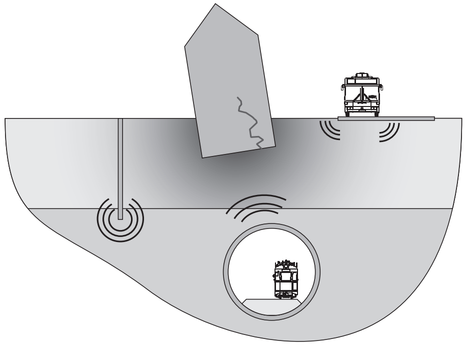
Prediction of railway induced vibrations in an urban environment

Pieter Coulier[1], Alice Cicirello[2], Geert Lombaert[1],
Geert Degrande[1], Hugh Hunt[2]

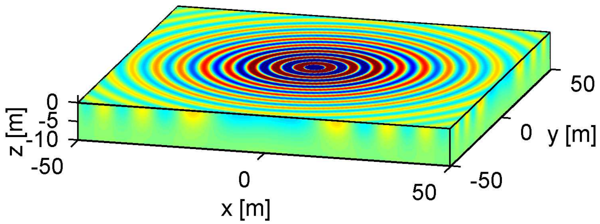
[1]KU Leuven, Department of Civil Engineering, Belgium
[2]University of Cambridge, Engineering Department, United Kingdom

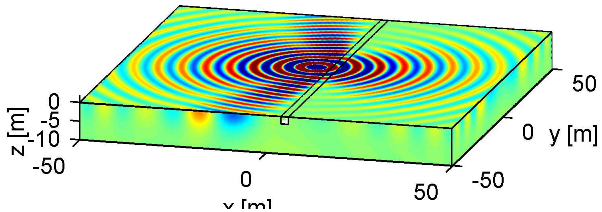
Railway induced vibrations

- Railway induced building vibrations may lead to ($1 - 80\,\mathrm{Hz}$):
  - malfunctioning of sensitive equipment ($\propto\ \mu\mathrm{m/s}$)
  - discomfort to people ($\propto\ 0.1\,\mathrm{mm/s}$)
  - damage to structures ($\propto\ 10\,\mathrm{mm/s}$)



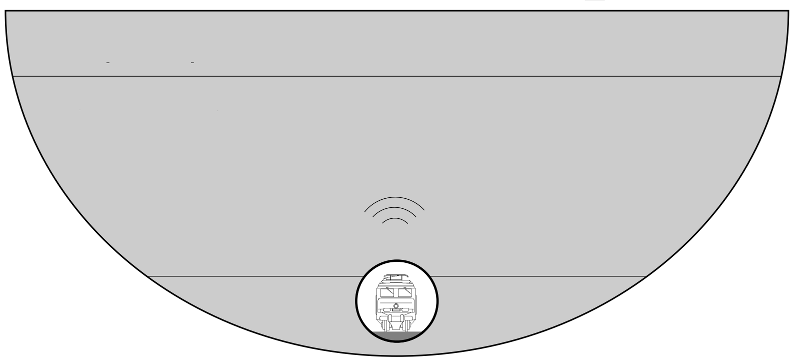
- Numerical prediction models are required for:
  - identifying and analysing problems of railway induced vibrations in new and existing situations
  - design of efficient mitigation measures (source, transmission path, and receiver)

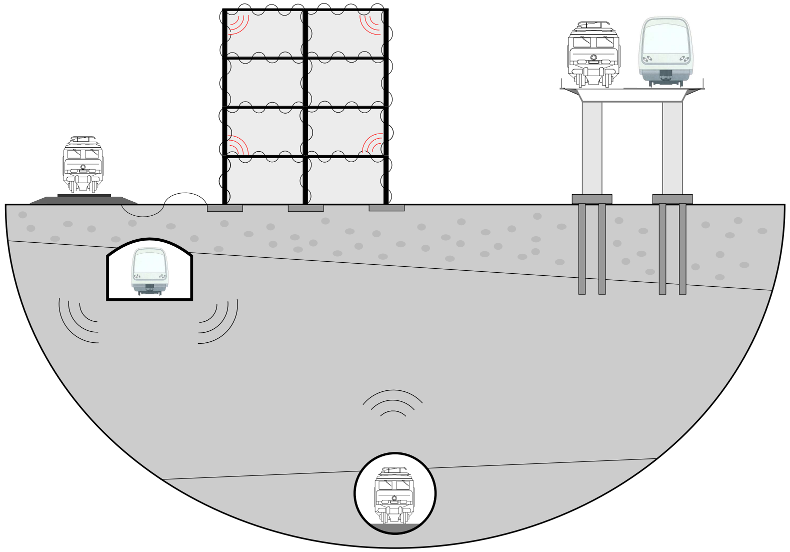




Railway induced vibrations

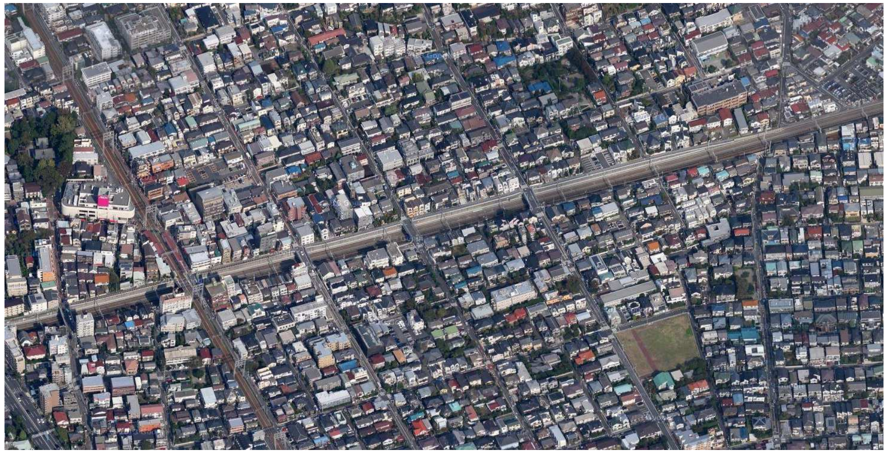
- Numerical models should account for:
  - vehicle–track interaction (static and dynamic train loads)
  - elastodynamic wave propagation in the soil
  - dynamic soil–structure interaction (SSI)
  - . . .
- Simplifications are often introduced in numerical models:
  - horizontally layered soil $\leftrightarrow$ soil inhomogeneities, inclined soil layers, . . .
  - perfect contact between soil and structures $\leftrightarrow$ voids at soil–structure interfaces
  - individual structures $\leftrightarrow$ urban environment
  - . . .



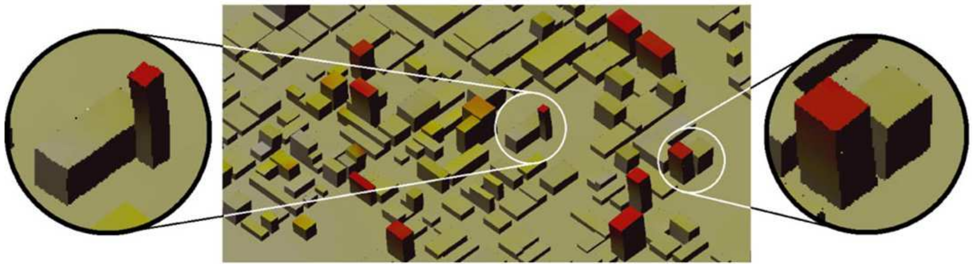


Urban environment

- Different buildings interact with each other due to interference of the scattered waves
  $\Rightarrow$ strongly coupled dynamic soil–structure interaction problems



- Studies of the *seismic city–site effect* [Wirgin and Bard, BSSA, 1996; Lombaert and Clouteau, JASA, 2009; . . .] are restricted to low frequencies ($< 10\,\mathrm{Hz}$)



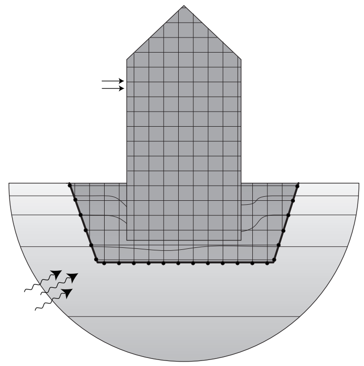
[Mazzieri et al., INT J NUMER METH ENG, 2013]

- The numerical prediction of railway induced vibrations involves frequencies up to $80\,\mathrm{Hz}$

Urban environment

- Coupled FE–BE methodology:
  - Structural domain(s): FE method

$$\left[\mathbf{K}_{\mathrm{b}} + i\omega\mathbf{C}_{\mathrm{b}} - \omega^2\mathbf{M}_{\mathrm{b}}\right]\underline{\hat{\mathbf{u}}}_{\mathrm{b}}(\omega) = \underline{\hat{\mathbf{f}}}_{\mathrm{b}}(\omega) + \underline{\hat{\mathbf{f}}}^{\mathrm{s}}_{\mathrm{b}}(\omega)$$



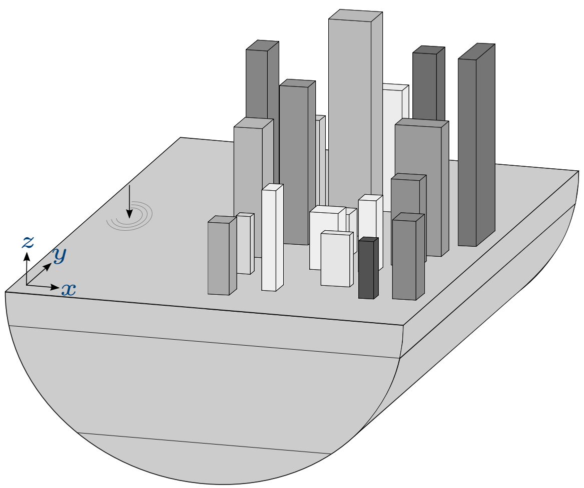
  - Soil domain: BE method

$$\widehat{\mathbf{U}}(\omega)\underline{\hat{\mathbf{t}}}(\omega) = \left[\widehat{\mathbf{T}}(\omega) + \mathbf{I}\right]\underline{\hat{\mathbf{u}}}(\omega)$$

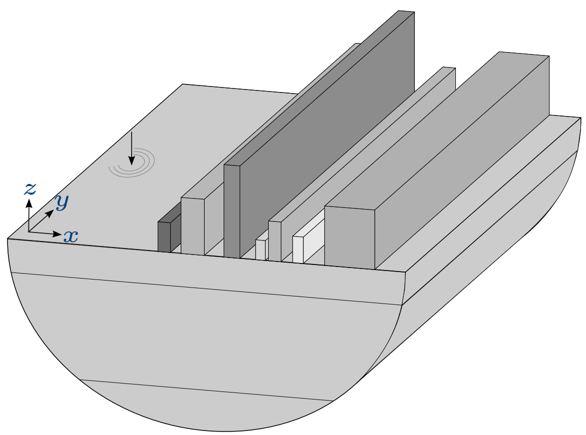
- 3D models:
  + complex geometries
  – computationally expensive

- 2.5D FE–BE models:
  + computationally efficient
  – invariant geometry

3D model



2.5D model



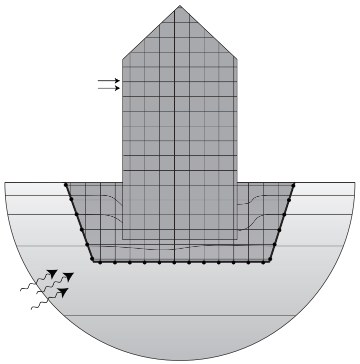
Urban environment

- Coupled FE–BE methodology:
  - Structural domain(s): FE method

$$\left[\mathbf{K}_{\mathrm{b}} + i\omega\mathbf{C}_{\mathrm{b}} - \omega^2\mathbf{M}_{\mathrm{b}}\right] \underline{\hat{\mathbf{u}}}_{\mathrm{b}}(\omega) = \underline{\hat{\mathbf{f}}}_{\mathrm{b}}(\omega) + \underline{\hat{\mathbf{f}}}^{\mathrm{s}}_{\mathrm{b}}(\omega)$$

  - Soil domain: BE method

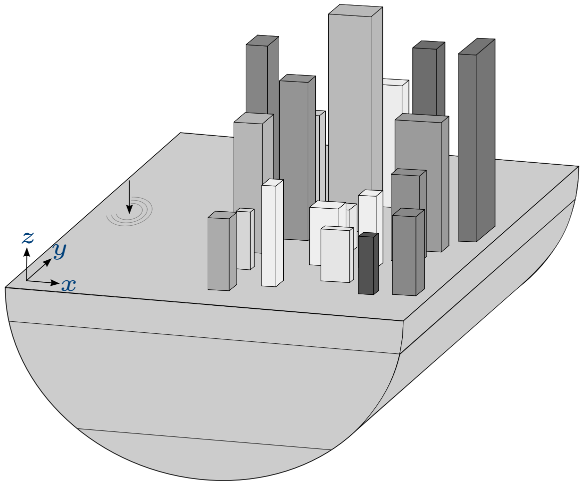
$$\widehat{\mathbf{U}}(\omega)\underline{\hat{\mathbf{t}}}(\omega) = \left[\widehat{\mathbf{T}}(\omega) + \mathbf{I}\right] \underline{\hat{\mathbf{u}}}(\omega)$$



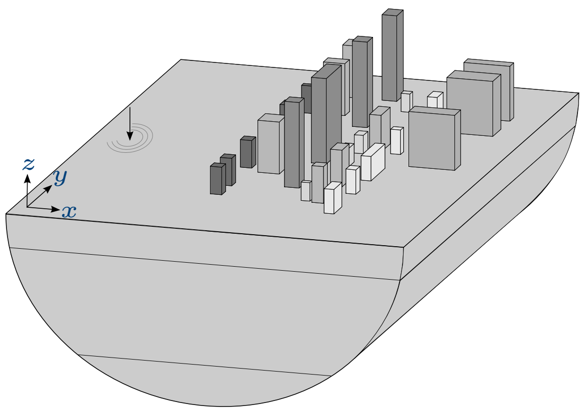
- 3D models:
  + complex geometries
  – computationally expensive

- 2.5D FE–BE models:
  + computationally efficient
  – invariant geometry

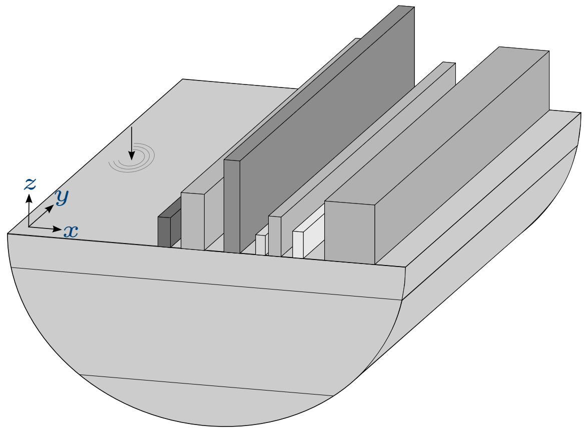
3D model



2.5D model with spatial windowing



2.5D model



2.5D FE–BE method

- Computationally efficient 2.5D approach in the frequency–wavenumber domain:
  - Forward Fourier transform from y to k_y:

$$\mathscr{F}\left[f(y), k_y\right] = \int_{-\infty}^{+\infty} f(y) \exp\left(i k_y y\right) \, \mathrm{d}y$$

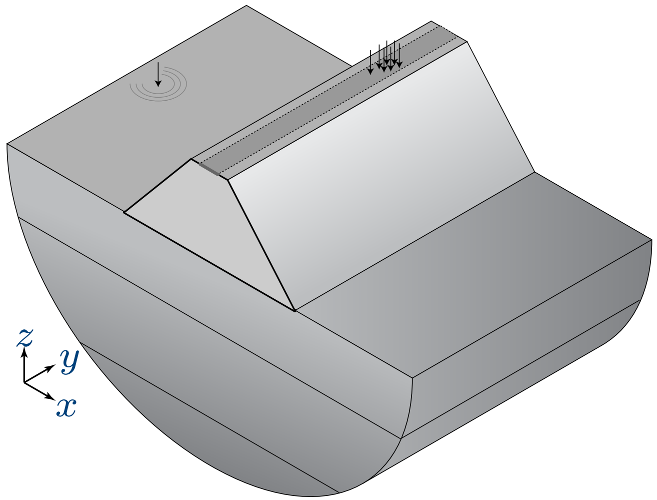
  - 2.5D FE–BE equations [François et al., COMPUT METHOD APPL M, 2010]:

$$\left[\widetilde{\mathbf{K}}_j(k_y,\omega) + i\omega\mathbf{C}_j - \omega^2\mathbf{M}_j\right] \underline{\tilde{\mathbf{u}}}_j(k_y,\omega) + \sum_{k=1}^{N} \widetilde{\mathbf{K}}^{\mathrm{s}}_{jk}(k_y,\omega)\underline{\tilde{\mathbf{u}}}_k(k_y,\omega) = \underline{\tilde{\mathbf{f}}}_j(k_y,\omega) + \underline{\tilde{\mathbf{f}}}^{\mathrm{s}}_j(k_y,\omega)$$

  - Inverse Fourier transform from k_y to y:

$$\mathscr{F}^{-1}\left[f(k_y), y\right] = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(k_y) \exp\left(-i k_y y\right) \, \mathrm{d}k_y$$

- (Dis)advantages?
  + computationally efficient
  – invariant geometry



2.5D FE–BE method

- Computationally efficient 2.5D approach in the frequency–wavenumber domain:
  - Forward Fourier transform from y to k_y:

$$\mathscr{F}\left[f(y), k_y\right] = \int_{-\infty}^{+\infty} f(y) \exp\left(ik_y y\right)\,\mathrm{d}y$$

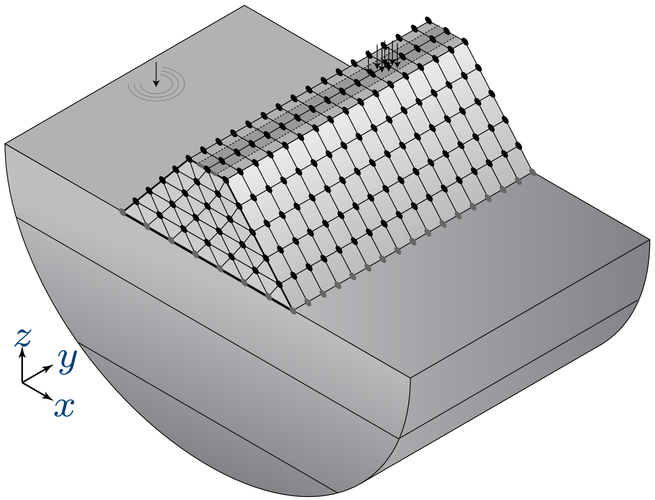
  - 2.5D FE–BE equations [François et al., COMPUT METHOD APPL M, 2010]:

$$\left[\widetilde{\mathbf{K}}_j(k_y,\omega) + i\omega\mathbf{C}_j - \omega^2\mathbf{M}_j\right]\underline{\tilde{\mathbf{u}}}_j(k_y,\omega) + \sum_{k=1}^{N}\widetilde{\mathbf{K}}^{\mathrm{s}}_{jk}(k_y,\omega)\underline{\tilde{\mathbf{u}}}_k(k_y,\omega) = \underline{\tilde{\mathbf{f}}}_j(k_y,\omega) + \underline{\tilde{\mathbf{f}}}^{\mathrm{s}}_j(k_y,\omega)$$

  - Inverse Fourier transform from k_y to y:

$$\mathscr{F}^{-1}\left[f(k_y), y\right] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(k_y) \exp\left(-ik_y y\right)\,\mathrm{d}k_y$$

- (Dis)advantages?
  + computationally efficient
  – invariant geometry



2.5D FE–BE method

- Computationally efficient 2.5D approach in the frequency–wavenumber domain:
  - Forward Fourier transform from y to k_y:

$$\mathscr{F}\left[f(y), k_y\right] = \int_{-\infty}^{+\infty} f(y) \exp\left(ik_y y\right)\, \mathrm{d}y$$

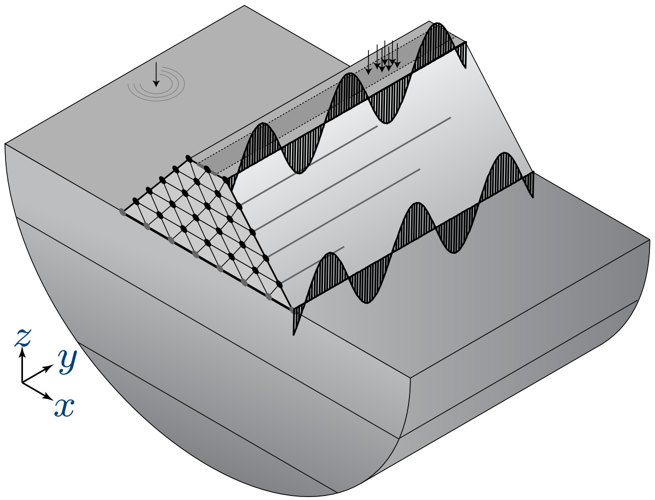
  - 2.5D FE–BE equations [François et al., COMPUT METHOD APPL M, 2010]:

$$\left[\widetilde{\mathbf{K}}_j(k_y,\omega) + i\omega\mathbf{C}_j - \omega^2\mathbf{M}_j\right]\underline{\tilde{\mathbf{u}}}_j(k_y,\omega) + \sum_{k=1}^{N}\widetilde{\mathbf{K}}^{\mathrm{s}}_{jk}(k_y,\omega)\underline{\tilde{\mathbf{u}}}_k(k_y,\omega) = \underline{\tilde{\mathbf{f}}}_j(k_y,\omega) + \underline{\tilde{\mathbf{f}}}^{\mathrm{s}}_j(k_y,\omega)$$

  - Inverse Fourier transform from k_y to y:

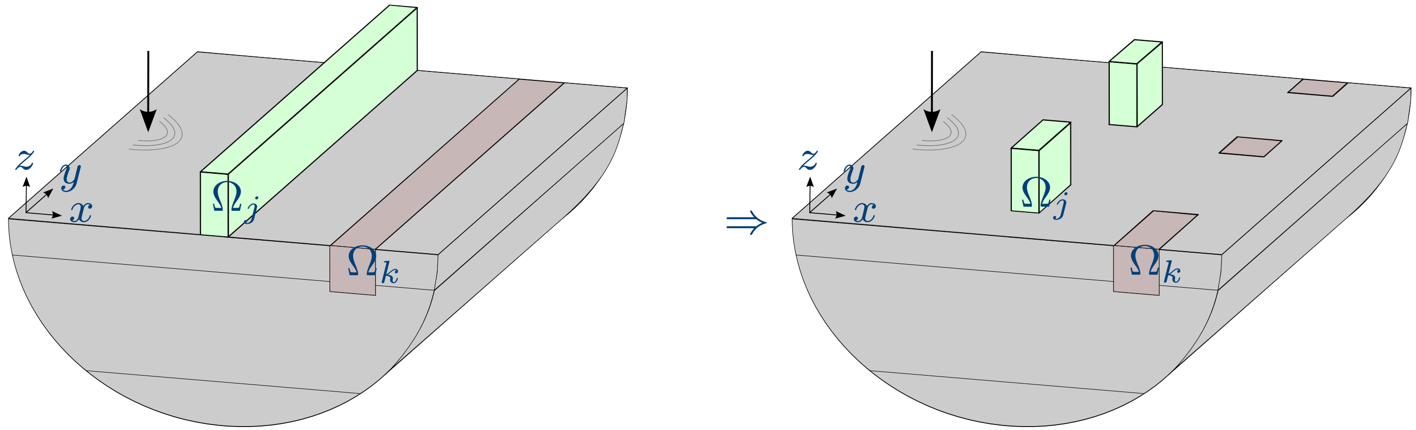
$$\mathscr{F}^{-1}\left[f(k_y), y\right] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(k_y) \exp\left(-ik_y y\right)\, \mathrm{d}k_y$$

- (Dis)advantages?
  + computationally efficient
  – invariant geometry



2.5D FE–BE method with spatial windowing

- Spatial windowing technique [Coulier et al., SOIL DYN EARTHQ ENG,2014]:
  - accounts for the presence of multiple structures of finite length along Ω_j
  - maintains the computational efficiency of a 2.5D approach



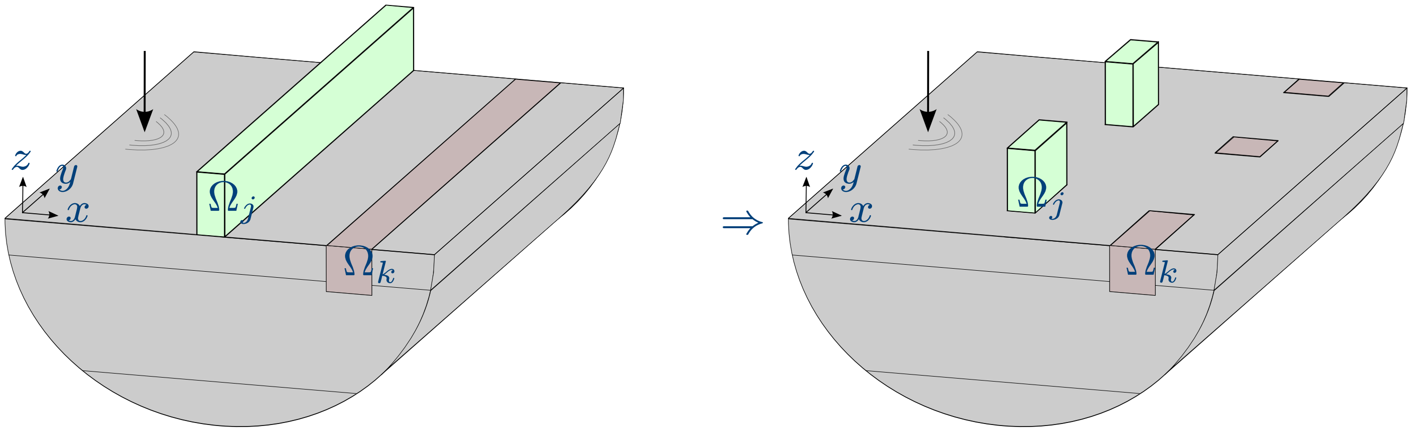
- Redistribution of the wavenumber spectrum over the entire wavenumber domain:

$$\underline{\tilde{\mathbf{u}}}_{\mathrm{sw},j}(k_y,\omega) = \sum_{i=1}^{n_j} \int_{y_{i_1}}^{y_{i_2}} \underline{\hat{\mathbf{u}}}_j(y,\omega) \exp\left(ik_y y\right) \, \mathrm{d}y = \ldots = \underline{\tilde{\mathbf{u}}}_j(k_y,\omega) * \tilde{w}_j\left(k_y\right)$$

⇒ spatial windowing technique only entails postprocessing of the original 2.5D results

2.5D FE–BE method with spatial windowing

- Spatial windowing technique [Coulier et al., SOIL DYN EARTHQ ENG,2014]:
  - accounts for the presence of multiple structures of finite length along Ω_j
  - maintains the computational efficiency of a 2.5D approach



- Redistribution of the wavenumber spectrum over the entire wavenumber domain:

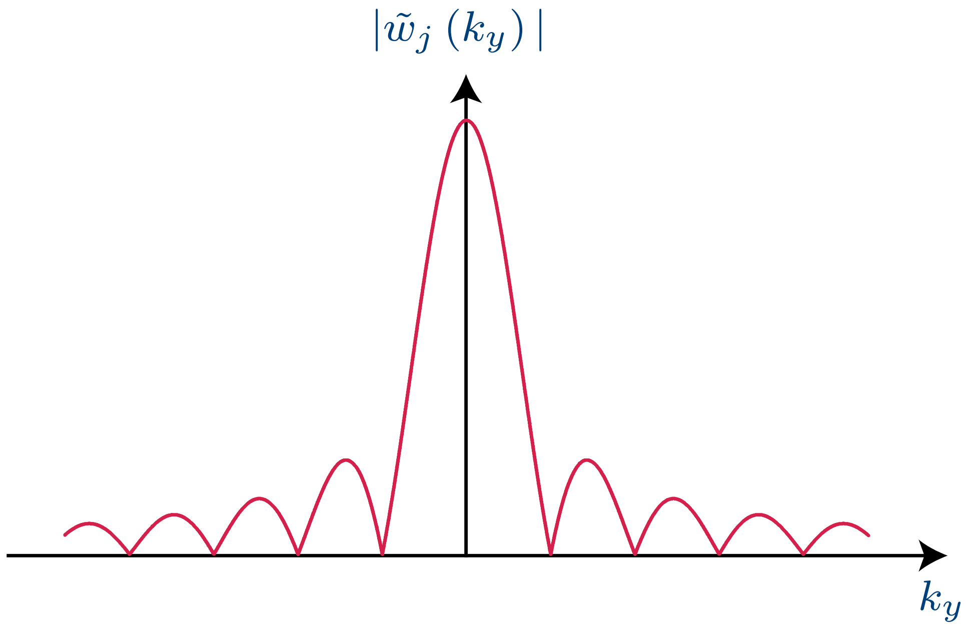
$$\underline{\tilde{\mathbf{u}}}_{\mathrm{sw},j}(k_y,\omega) = \sum_{i=1}^{n_j} \int_{y_{i_1}}^{y_{i_2}} \underline{\hat{\mathbf{u}}}_j(y,\omega) \exp\left(ik_y y\right) \mathrm{d}y = \ldots = \underbrace{\underline{\tilde{\mathbf{u}}}_j(k_y,\omega)}_{\text{2.5D results}} * \underbrace{\tilde{w}_j\left(k_y\right)}_{\text{windowing}}$$

⇒ spatial windowing technique only entails postprocessing of the original 2.5D results

2.5D FE–BE method with spatial windowing

- Windowing function $\tilde{w}_j\left(k_y\right)$:

$$\tilde{w}_j\left(k_y\right) = \sum_{i=1}^{n_j} \int_{y_{i_1}}^{y_{i_2}} \frac{1}{2\pi} \exp\left(ik_y y\right) \mathrm{d}y = \sum_{i=1}^{n_j} \left[\frac{1}{2\pi} \frac{\exp\left(ik_y y_{i2}\right)}{ik_y} \left(1 - \exp\left[-ik_y L_{yi}\right]\right) \right]$$



- Spatial windowing of Ω_j also effects all other structures Ω_k $(k \neq j)$
  $\Rightarrow$ variables should be recalculated, resulting in an iterative procedure

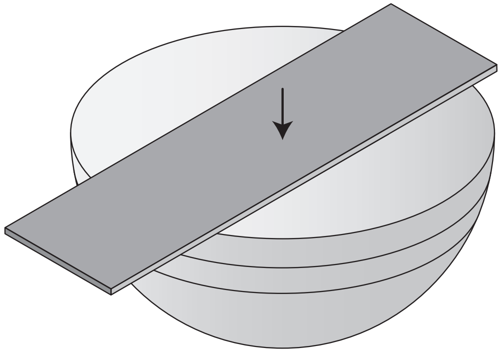
2.5D FE–BE method with spatial windowing

- Example (single structure):
  - Square concrete surface foundation
    ($5\,\mathrm{m} \times 5\,\mathrm{m} \times 0.25\,\mathrm{m}$, $E = 33\,\mathrm{GPa}$, $\nu = 0.20$, $\rho = 2500\,\mathrm{kg/m^3}$)
  - Layered halfspace

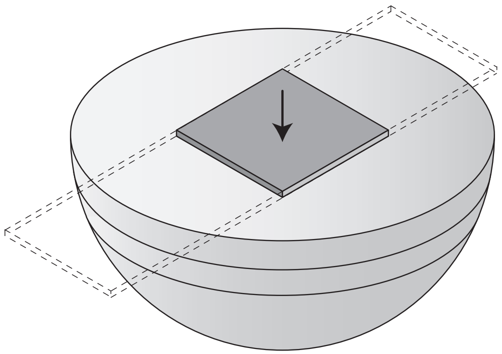
Layer	Thickness $[\mathrm{m}]$	C_s $[\mathrm{m/s}]$	C_p $[\mathrm{m/s}]$	$\beta_\mathrm{s} = \beta_\mathrm{p}$ $[-]$	ρ $[\mathrm{kg/m^3}]$
1	2	150	300	0.025	1800
2	2	250	500	0.025	1800
3	∞	300	600	0.025	1800

  - Unit harmonic vertical point excitation at the center of the foundation

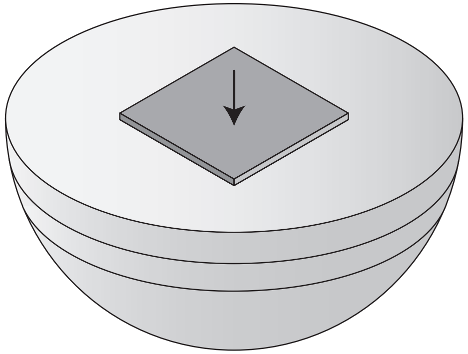
2.5D model



2.5D model
with spatial windowing

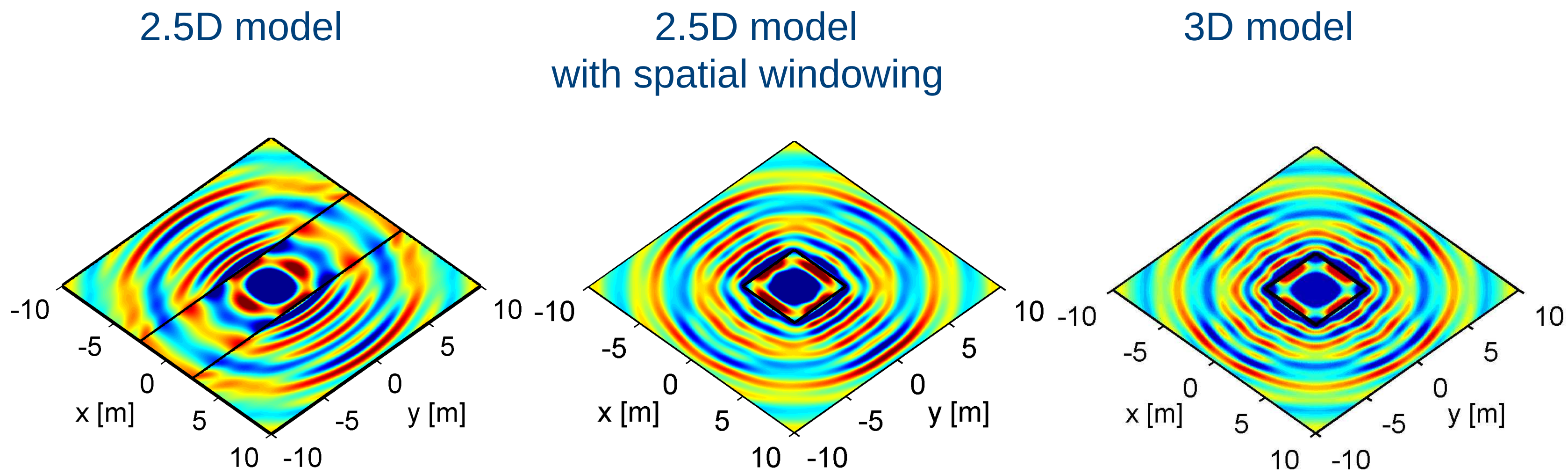


3D model

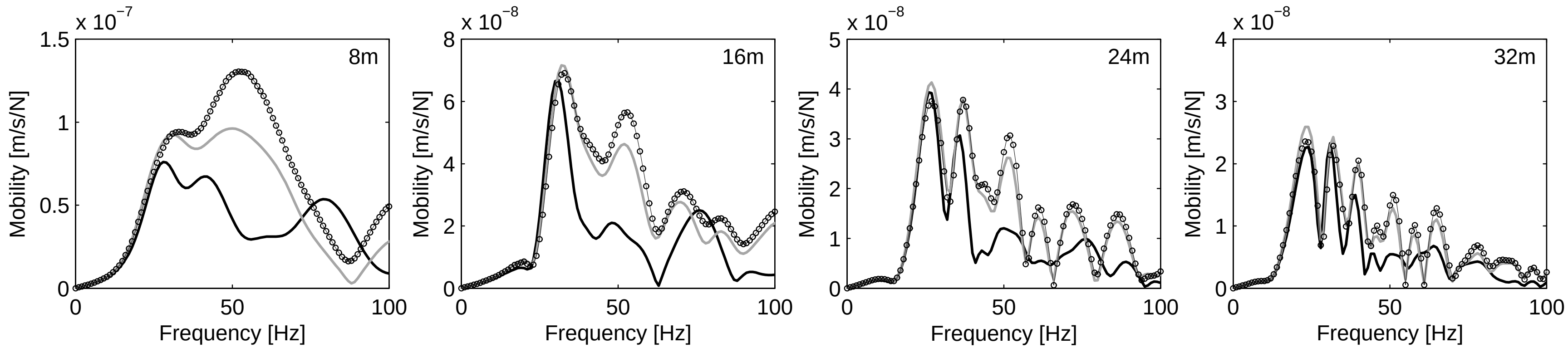


2.5D FE–BE method with spatial windowing

- Vertical displacement $\hat{u}_z(\mathbf{x},\omega)$ at $100\,\mathrm{Hz}$



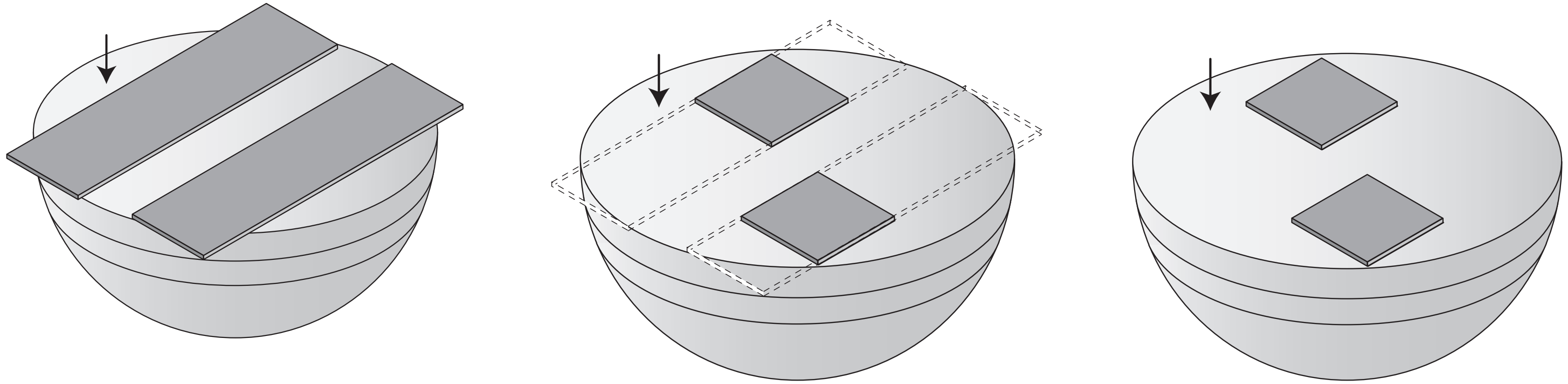
- Free field mobility along the line $y = 0\,\mathrm{m}$, calculated with a 2.5D FE–BE model (black line), a 2.5D FE–BE model with spatial windowing (circles), and a 3D FE–$\mathscr{H}$-BE model (grey line).



2.5D FE–BE method with spatial windowing

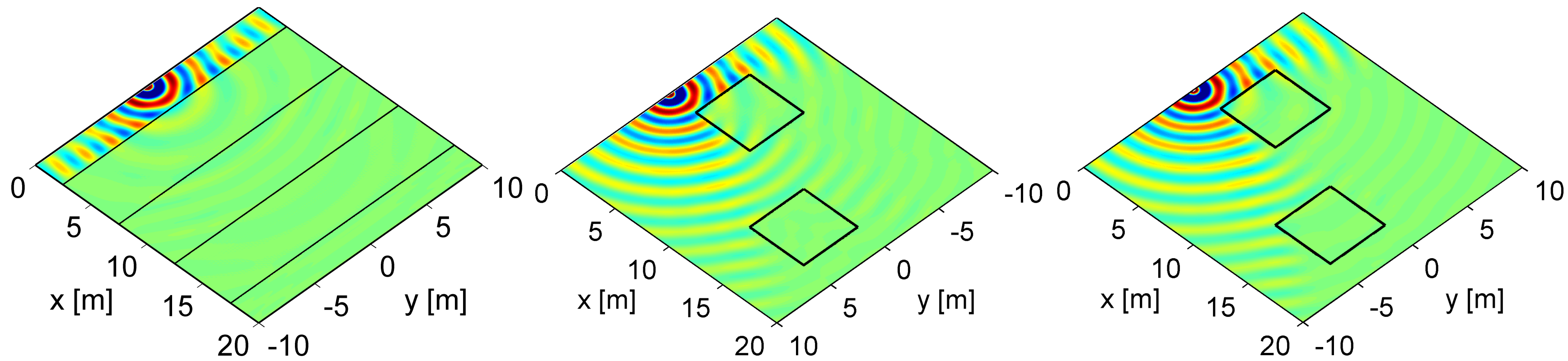
- Example (multiple structures):
  - Unit harmonic vertical point excitation at the surface of a layered halfspace

2.5D model | 2.5D model with spatial windowing | 3D model



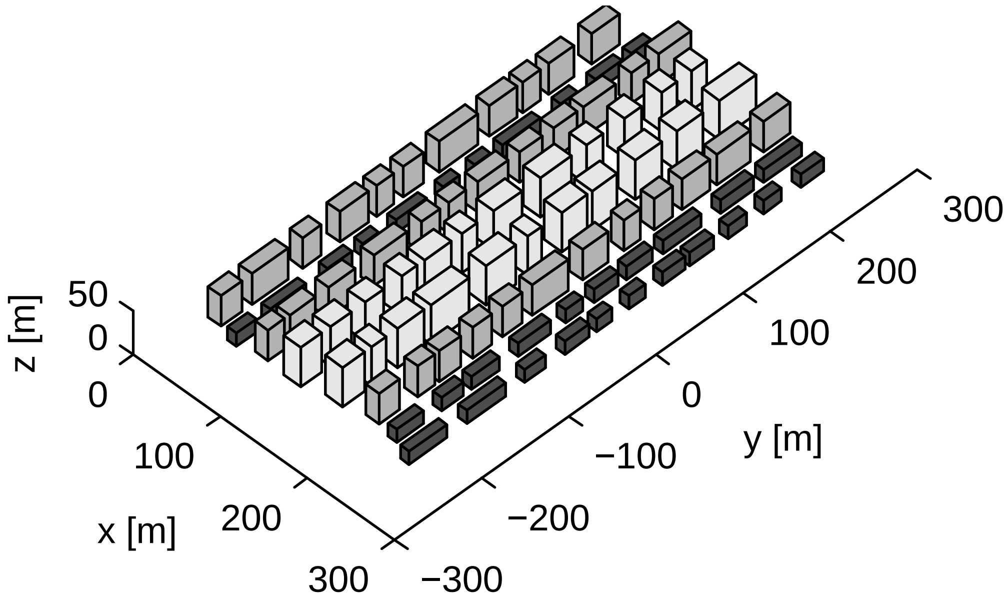
  - Vertical displacement $\hat{u}_z(\mathbf{x}, \omega)$ at $100\,\mathrm{Hz}$

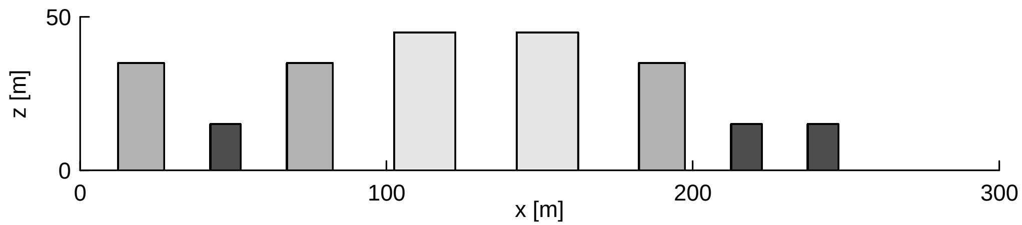
2.5D model | 2.5D model with spatial windowing | 3D model



City model

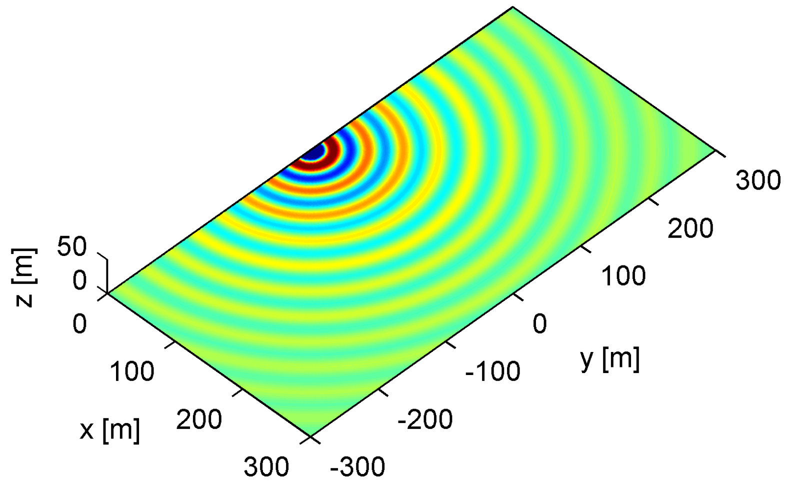
- Case study: city–quarter of 90 non-equally spaced, non-equally sized buildings
  - homogeneous solid blocks on top of a concrete surface foundation
  - the building lengths L_y and the interbuilding spacings Δ_y are generated from uniform distributions $\mathcal{U} \in [15\,\mathrm{m}, 45\,\mathrm{m}]$ and $\mathcal{U} \in [5\,\mathrm{m}, 25\,\mathrm{m}]$.
  - homogeneous halfspace
- Unit harmonic vertical point excitation at the surface of the halfspace



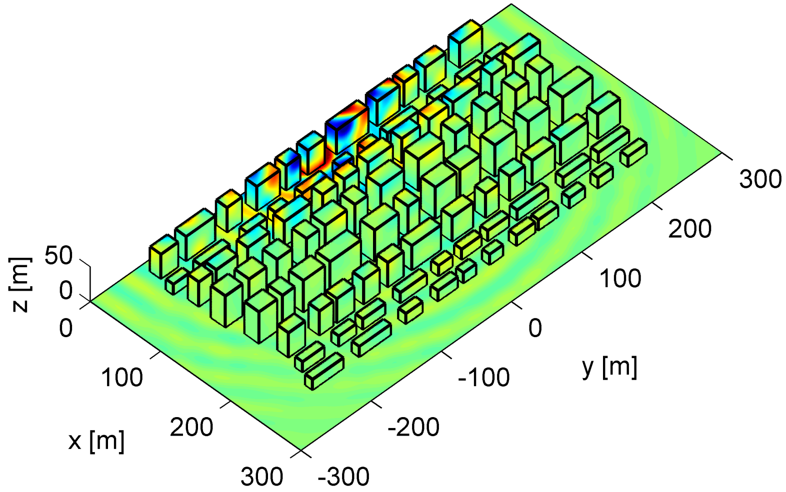


Results: displacements

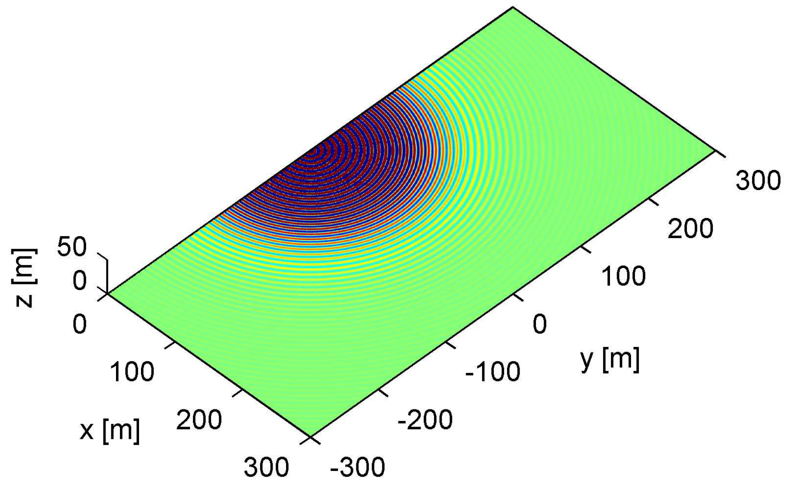
- Vertical displacement $\hat{u}_z(\mathbf{x}, \omega)$ at $5\ \mathrm{Hz}$
  - incident wavefield



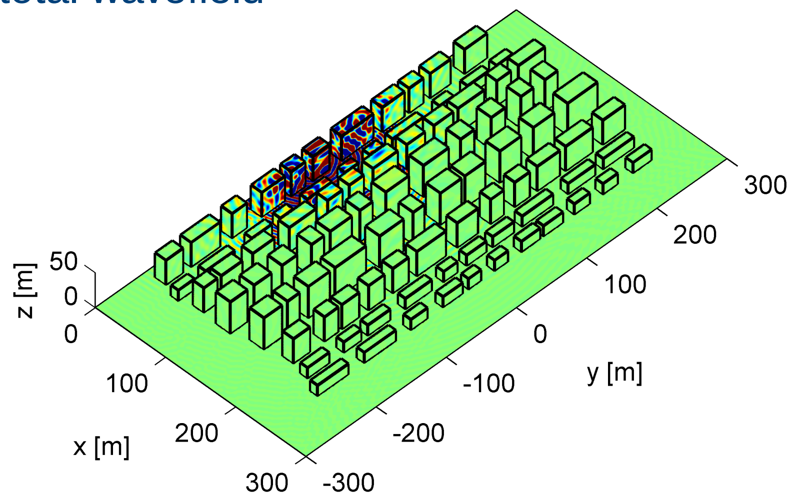
  - total wavefield



- Vertical displacement $\hat{u}_z(\mathbf{x}, \omega)$ at $25\ \mathrm{Hz}$
  - incident wavefield



  - total wavefield



Results: power flow

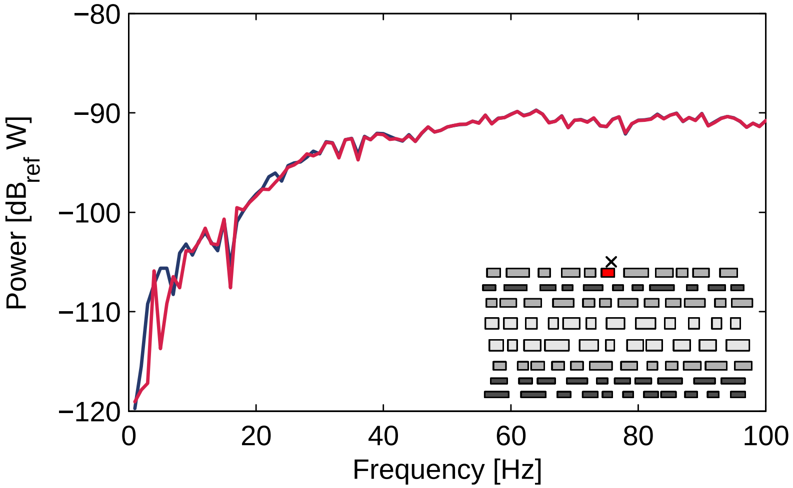
- The mean power flow $\left\langle \hat{P}(\Gamma,\omega)\right\rangle$ through a surface Γ is calculated as:

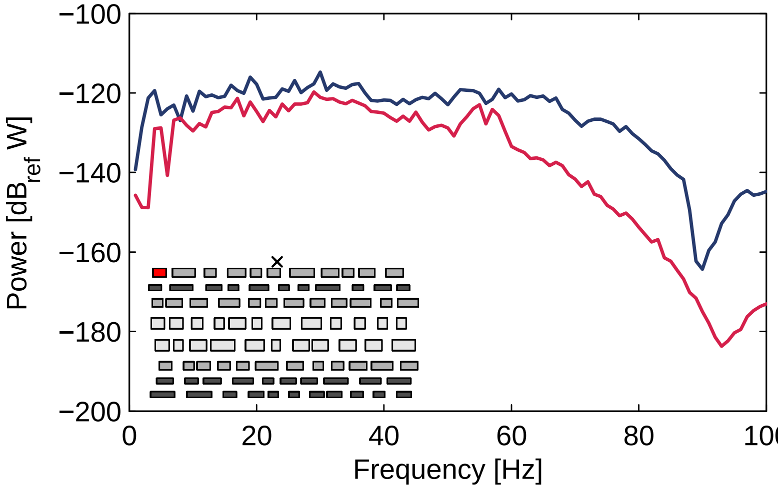
$$\left\langle \hat{P}(\Gamma,\omega)\right\rangle = \int_{\Gamma} \langle \hat{p}^{\mathbf{n}}(\omega)\rangle \; d\Gamma$$

with

$$\langle \hat{p}^{\mathbf{n}}(\omega)\rangle = -\frac{1}{2}\mathrm{Re}(\hat{\mathbf{t}}^{\mathbf{n}}(\omega)^{\star} \cdot \hat{\mathbf{v}}(\omega)) = -\frac{1}{2}\mathrm{Re}(i\omega\hat{\mathbf{t}}^{\mathbf{n}}(\omega)^{\star} \cdot \hat{\mathbf{u}}(\omega))$$

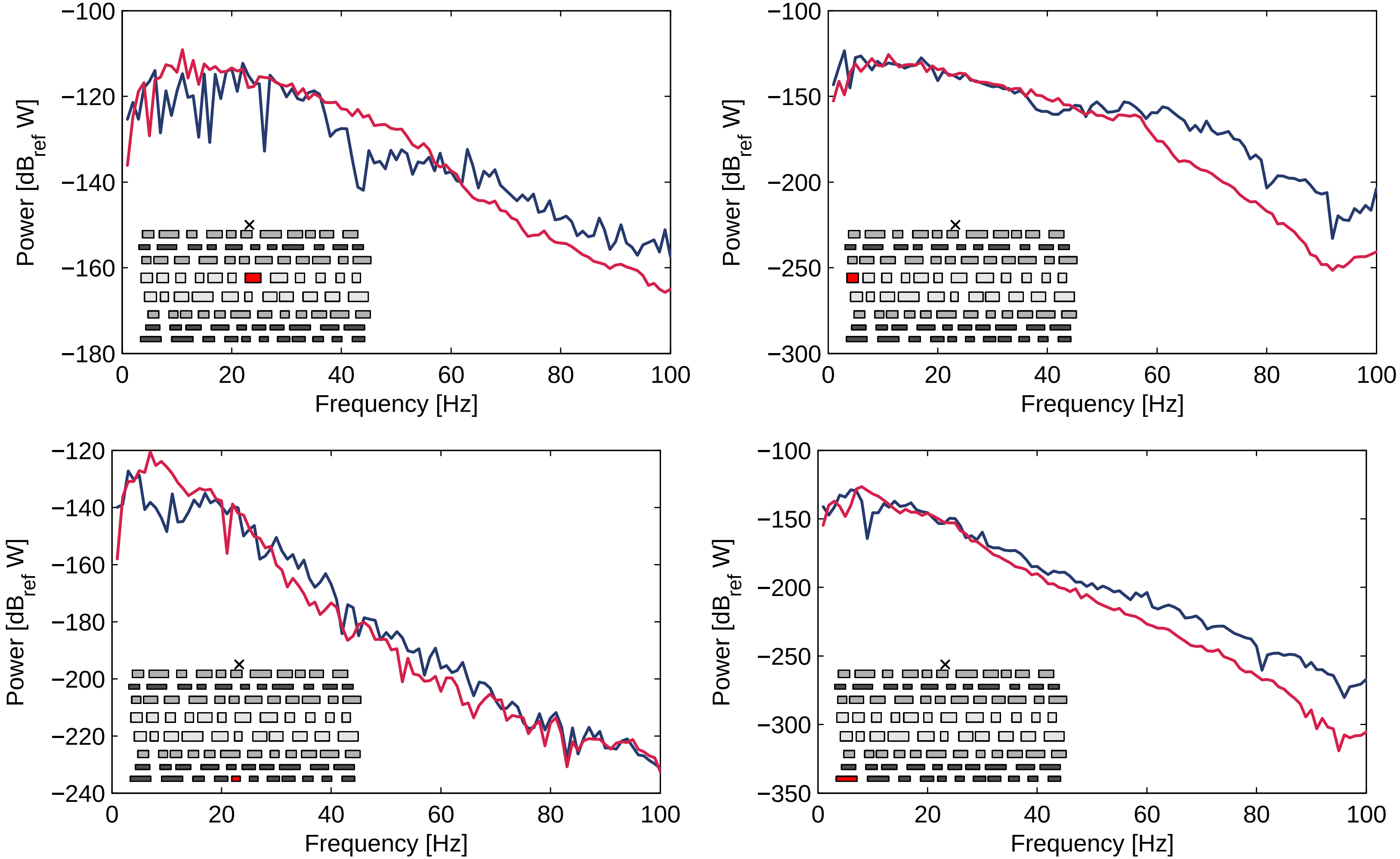
- Net power input into the building if the presence of the surrounding buildings is **disregarded (red line)** or **taken into account (blue line)**





Results: power flow

- Net power input into the building if the presence of the surrounding buildings is **disregarded (red line)** or **taken into account (blue line)**



Conclusions

- Spatial windowing: enables the application of 2.5D FE–BE models, even if the assumption of longitudinal invariance is not fulfilled
- Urban environment: accounting for the presence of the surrounding buildings affects the structural response

Outlook

- Monte Carlo simulations: probabilistic assessment of the interaction effects
- City $\rightarrow$ equivalent top soil layer (cfr. [Boutin and Roussillon, INT J ENG SCI, 2006]) ?